Teacher notes
Topic C

Sphere oscillating inside a bowl.

Consider a spherical bowl of radius R and a sphere of radius r inside the bowl. The sphere is displaced slightly from its equilibrium position and released. The sphere rolls without slipping inside the bowl.

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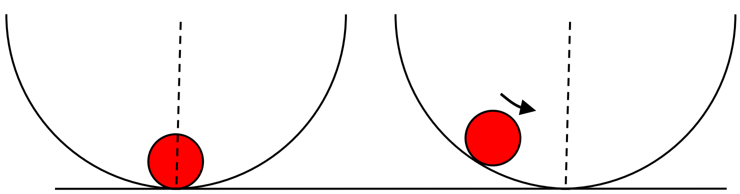
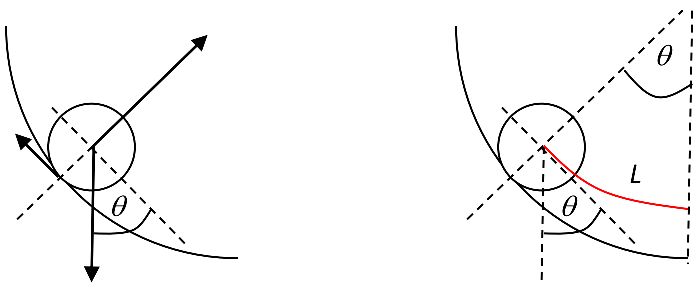


(a) Show that for very small displacements from equilibrium the sphere will perform simple harmonic oscillations with period given by $T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$.

(b) In an experiment, students used the same bowl and balls of different radius in order to verify the relationship above. They plotted r versus T^2.
   (i) Explain why they will obtain a straight line.
   (ii) Determine the slope of the straight line.
   (iii) Discuss the best way to obtain the value of R from the graph.

Answers

(a)



Friction provides the torque:

$fr = I\alpha = I\frac{a}{r} \Rightarrow f = \frac{Ia}{r^2}$. (We have rolling without slipping so $\alpha = \frac{a}{r}$.)

Newton's second law:

$$mg\sin\theta - f = ma$$
$$mg\sin\theta - \frac{Ia}{r^2} = ma$$
$$a = \frac{g\sin\theta}{1+\frac{I}{mr^2}}$$

For a sphere, $I = \frac{2}{5}mr^2$ and so $a = \frac{g\sin\theta}{1+\frac{2}{5}} = \frac{5g\sin\theta}{7}$. This acceleration is opposite to the displacement. This is the tangential acceleration which brings the ball back towards equilibrium. The displacement is the length of the red arc of length $L = (R-r)\theta$. If the displacement is small, the angle θ is small and so $\sin\theta \approx \theta$, i.e.,

$$a = \frac{5g\sin\theta}{7} \approx \frac{5g\theta}{7} = \frac{5g}{7}\frac{L}{R-r}.$$

So, the acceleration is opposite to displacement and proportional to it. This means that we have SHM with $\omega^2 = \frac{5g}{7(R-r)}$. Hence $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{7(R-r)}{5g}}$.

(b)

(i)

$$T^2 = 4\pi^2 \frac{7(R-r)}{5g} = \frac{28\pi^2}{5g}(R-r)$$

$$R - r = \frac{5gT^2}{28\pi^2}$$

$$r = R - \frac{5g}{28\pi^2}T^2$$

$$y = R - \frac{5g}{28\pi^2}x$$ which is the equation of a straight line

(ii) The gradient is $-\frac{5g}{28\pi^2}$ (about – 0.18 m s^{-2}).

(iii) The vertical axis intercept is *R*. This is the best way to find *R* since it uses the line of best fit of all the data.